# FORMATION OF DIFFERENTIAL EQUATION

The formation of differential equation gives the relationship between the different order differential coefficients.

Q1) If $y = \frac{ax-b}{a-bx}$, then prove that

$$2y_1y_3 = 3y_2^2$$

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$y\frac{1}{2}$

Solution:

Given: $y = \frac{ax-b}{a-bx}$

$\therefore (a-bx)y = (ax-b)$

On differentiating we get $\cancel{(a-bx)y_1 + y(-b) = a}$

$\cancel{-b(a-bx)y_1 = a}$.

~~On again differentiating we get:~~

$$\cancel{(a-bx)y_1 + y(}$$

$$(a-bx)y_1 + y(-b) = a$$

$$(a-bx)y = (ax-b)$$

or, $$(a-bx)y_1 + y(-b) = a$$

$$(a-bx)y_1 - by = a$$

or $$(a-bx)y_1 = a + by$$

On again differentiating we get:

$$(a-bx)y_2 + y_1(-b) = 0 + by_1$$

or, $$(a-bx)y_2 - by_1 = by_1$$

$$(a-bx)y_2 = 2by_1 \quad \text{—— (i)}$$

Now,

On again differentiating we get:

$$(a-bx)y_3 + (-b)y_2 = 2by_2$$

$$(a-bx)y_3 - by_2 = 2by_2$$

$$\Rightarrow (a-bx)y_3 = 2by_2 \; 3by_2$$

$$(a-bx)y_3 = 3by_2 \quad \text{—— (ii)}$$

On dividing eqn (i) by (ii) we get:

$$\frac{\cancel{(a-bx)}y_2}{\cancel{(a-bx)}y_3} = \frac{2by_1}{3by_2}$$

$$\therefore 3\cancel{b}y_2 y_2 = 2\cancel{b}y_1y_3$$

$$\therefore 3y_2^2 = 2y_1y_3$$ proved

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Q) If $y = e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_2 - xy_1 = a^2y$

Solution :- $y = e^{a\sin^{-1}x}$ —— (i)

on differentiating we get

$$y_1 = e^{a\sin^{-1}x} \cdot a \frac{1}{\sqrt{1-x^2}}$$

$$y_1 = \frac{a}{\sqrt{1-x^2}} \cdot e^{a\sin^{-1}x}$$

or, $y_1 = \frac{a}{\sqrt{1-x^2}} \cdot y$ [from (i)]

$(y_1\sqrt{1-x^2}) = ay$ —— (ii)

Now, squaring on the both sides we gets.

$$y_1^2(1-x^2) = a^2y^2$$

On again differentiating we get:

$$(1-x^2) \cdot 2y_1y_2 + y_1^2(-2x) = a^2 \cdot 2y \cdot y_1$$

or, $(1-x^2)y_2 - y_1x = a^2y$

proved.

Q If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$

Solution: Given:

$y = \sin(m\sin^{-1}x)$ —— (i)

On differentiating we get

$$y_1 = \cos(m\sin^{-1}x) \cdot m \cdot \frac{1}{(\sqrt{1-x^2})}$$

or $y_1(\sqrt{1-x^2}) = m \cdot \cos(m\sin^{-1}x)$

Now, On squaring on the both sides we get.

[Note. $\frac{d}{dx}(\sqrt{x^2-1}) = \frac{1}{2\sqrt{x^2-1}}$]

$y_1^2 (1-x^2) = m^2 \cos^2(m.\sin^{-1}x)$

$= m^2 [1 - \sin^2(m.\sin^{-1}x)]$

$y_1^2 (1-x^2) = m^2 [1 - y^2]$

Now, on again differentiating we get

$(1-x^2) 2y_1 y_2 + y_1^2(-2x) = 0 - m^2 2 y y_1$

also, $(1-x^2) y_2 - x y_1 = - m^2 y$ proved

## DETERMINATION OF $n$th derivative of Algebraic rational Function.

In order to find out the $n$th derivative of any rational function, we should first find out the partial fractions of that function.

Partial fraction are two forms:

i) One form is that in which the denominator contains linear factors only.

ii) The other form in which the denominator contains linear factor only. as well as quadratic factors.

[If the denominator in the partial fraction consists of linear (first degree) expression then its $n$th derivative can be found by direct application of formula. But the expression in the denominator consist of an second degree, then $n$th derivative is found by the application of De Moivre's Theorem.

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Case-I : When the denominator of the given expression contains only real linear factors.

~~Solutions~~

Q) If $y = \frac{x+3}{(x+1)(x+2)}$, find $y_n$.

Solution: $y = \frac{x+3}{(x+1)(x+2)}$

$$\frac{x+3}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$x+3 = \frac{A(x+2) + B(x+1)}{(x+1)(x+2)}$$

put $x = -2$

$$-2+3 = 0 + B(-2+1)$$

$$1 = -B$$

$$\therefore B = -1$$

Similarly

$$A = 2.$$

$$\therefore \frac{x+3}{(x+1)(x+2)} = \frac{2}{(x+1)} - \frac{1}{(x+2)}$$

$\because$ we know that,

$$y = \frac{1}{(ax+b)} = \frac{a^n(-1)^n n!}{(ax+b)^{n+1}}$$

$$\therefore y = \frac{2\cdot(-1)^n\cdot n!}{(x+1)^{n+1}} - \frac{1\cdot(-1)^n n!}{(x+2)^{n+1}}.$$

$$= (-1)^n \cdot n! \left[\frac{2}{(x+1)^{n+1}} - \frac{1}{(x+2)^{n+1}}\right] \text{ Ans.}$$